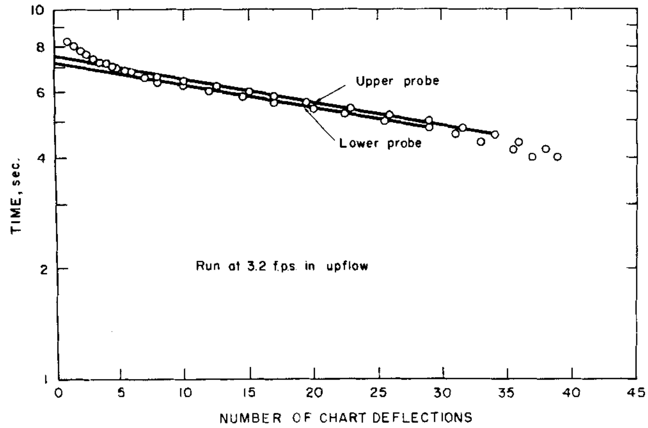

Fig. 3. Treatment of electrical conductivity curves.

was noted on the first probe vs. the chart deflection. When this plot is made, the tail of the curve falls on a straight line, with the exception of the very end of the curve. The straight line is drawn with the last points falling off the line and extrapolated to zero deflection. This treatment approximated the tail of the curve very well even though we can not justify it theoretically (see Figure 3). Typical dispersion coefficients calculated using this tail of curve approximation are shown in Table 1. Complete data tables may be found in reference *1*.

We believe that the measurement technique could be of value in studying two-phase systems.

TABLE 1. LIQUID PHASE AXIAL DISPERSION COEFFICIENTS IN UPFLOW

Void fraction	Avg. liquid velocity ft./sec.	axial dispersion coef. sq.ft./sec.
10%	6.2	2.0
20%	6.2	2.0
30%	6.2	2.7
10%	3.2	0.23
20%	3.2	0.18
30%	3.2	0.30

ACKNOWLEDGMENT

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NOTATION

σ^2 = second moment about the mean
N_{Pe} = Peclet Number

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Some Remarks on the Stability of Parallel of Non-Newtonian Fluids

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STABILITY OF PLANE POISEUILLE FLOW

The part played by viscoelasticity in contributing towards drag reduction and related phenomena in the turbulent flow of viscoelastic liquids has been investigated experimentally by many authors. The concensus of opinion is that viscoelasticity is a dominant factor, while the variation of apparent viscosity with rate of shear assumes a secondary role. Accordingly, a stability analysis was carried out by Chan Man Fong and Walters (*1*) for plane Poiseuille flow to determine what effect elasticity had on the stability criterion. They found that, according to infinitesimal disturbance theory, elasticity destabilizes the flow. This appears to be in agreement with some experimental results (*2* to *4*), but not with others (*5*, *6*). In fact, extreme values of the critical Reynolds number have been found to range from 1 to 6,000.

One possible reason for the apparent discrepancy between the various experimental studies may be that the non-Newtonian viscosity of the elastic liquids has a larger effect than was supposed in the case of the more concentrated polymer solutions.

In order to clarify this point, a stability analysis was carried out (*7*) for a third-order fluid which exhibits a variation in viscosity with rate of shear in simple shear. The analysis of Chan Man Fong and Walters (*1*), was carried out for what was essentially a second-order fluid in the sense of Coleman and Noll (*8*), and the extension of this work to the third-order fluid would appear to be the next step towards resolving the present dichotomy (*9*). Some justification for using the order constitutive equations in what is called an *unsteady* flow problem can be obtained a posteriori from a detailed comparison of

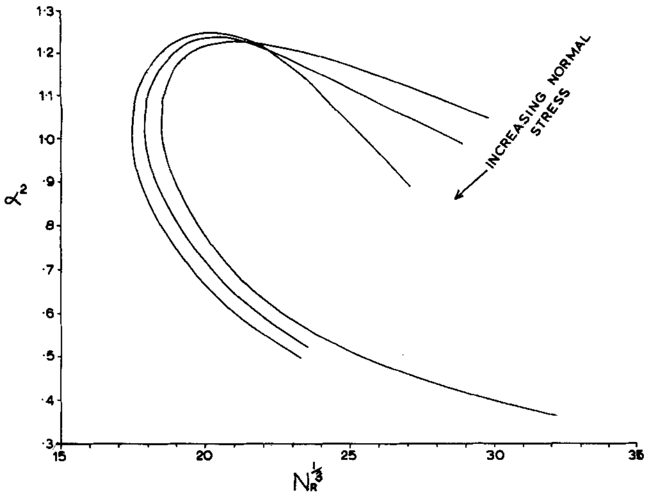

Fig. 1. Neutral stability curves for plane poiseuille flow.

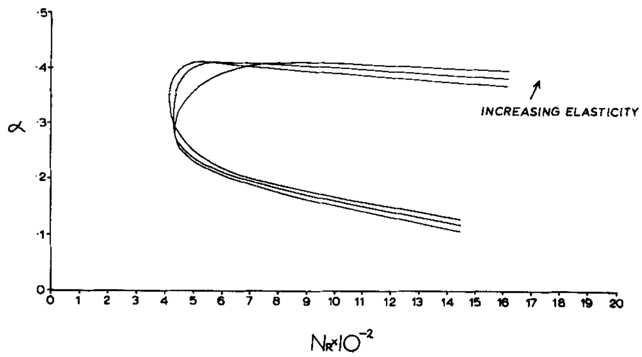

Fig. 3. Neutral stability curves for boundary-layer flow.

the results of the present investigation for a third-order fluid with those of Chan Man Fong and Walters (*1*) for a second-order fluid.

The effect of the extra order on the critical Reynolds number, below which the flow is stable, has been investigated by the usual normal-mode stability analysis, using an extension of Michael's method (*10*). The details of the lengthy analysis are omitted, since the method is adequately described elsewhere (*1, 10*).

We illustrate the results of the analysis by means of two figures. In Figure 1, the viscosity is kept constant, and the normal stress differences are allowed to vary; and in Figure 2, the normal stresses are kept constant and allowance is made for a decrease in apparent viscosity. In this way, it is possible to isolate the effects of normal stress differences and viscosity variation.

In keeping with the usual procedure, we have plotted $N_R^{1/3}$ against α^2, where N_R is the Reynolds number and α is the wave number. The Reynolds number is based on the limiting viscosity at small rates of shear. However, if a Reynolds number is used based on the conditions which prevail at the onset of instabilities, the conclusions are the same (*7*).

The important point on the neutral stability curve is that where the Reynolds number has a minimum value (called the *critical Reynolds number* N_{RC}). It will be observed from Figure 1 that N_{RC} decreases as the magnitude of the normal stress parameter increases. Hence, the normal stresses have a destabilizing effect on the flow. This result is in agreement with that of Chan Man Fong and Walters (*1*).

Figure 2 shows that the critical Reynolds number also decreases as the viscosity departs more and more from a constant value, so that the shear-thinning properties of the fluid also tend to destabilize the flow.

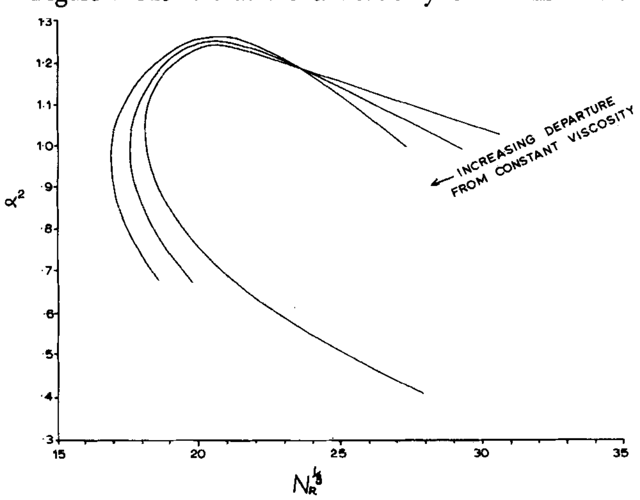

Fig. 2. Neutral stability curves for plane poiseuille flow.

We see, therefore, that the introduction of a variation in viscosity effect of the third-order type will not account for any of the observed experimental results which suggest stabilization in channel flow.

STABILITY OF BOUNDARY-LAYER FLOW

An analysis has also been carried out to determine the effect of elasticity on the stability of a laminar boundary layer. In this case, consideration has been confined to the second-order fluid. There are two reasons for undertaking such a study.

1. There is strong experimental evidence to support the stabilization of the boundary layer (*11, 12*) in contrast to the situation which exists in channel flow where either stabilization or destabilization may occur.
2. There is more justification for using the linear perturbation analysis in the case of boundary-layer flows, since such an analysis leads to close agreement with experiment in the case of Newtonian liquids (*13*).

Figure 3 illustrates the results of the analysis. It can be observed that the presence of elasticity raises the critical Reynolds number. (Elasticity in the context of the second-order fluid manifests itself only through normal-stress differences). Thus elasticity appears to have a stabilizing effect on the flow. To obtain some idea of the magnitude of this effect, we increased a suitable elastic parameter from 0 to 0.1 which resulted in a 5% increase in the critical Reynolds number.

It is interesting to note that the boundary-layer analysis leads to stabilization in contrast to the plane Poiseuille problem. It is also interesting to note that the theoretical prediction is in agreement with experimental observation.

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